

AdS/CFTのコライダー 物理への応用

八田佳孝
(筑波大数理)

2010/9/12 日本物理学会秋季大会 (九州工大)

Why AdS/CFT?

- Perturbative QCD very successful for hard processes. Why bother AdS?
- High CM energy does not mean weak coupling. There are many unsolved problems
- Hope AdS/CFT can bring new insights into the soft aspects of high energy QCD. Certain features may be universal.

Collider experiments

LEP (CERN, 1989~2000)

electron-positron annihilation $\sqrt{s} < 200\text{GeV}$

HERA (DESY, 1990~2007)

Deep inelastic scattering (DIS) $\sqrt{s} \approx 320\text{GeV}$

Tevatron (Fermilab, 1987~)

proton-antiproton $\sqrt{s} = 1.96\text{ TeV}$

RHIC (Brookhaven, 2000~)

proton-proton
nucleus-nucleus, $\sqrt{s_{NN}} \approx 200\text{ GeV}$

LHC (CERN, 2008~)

proton-proton, $\sqrt{s} = 900\text{ GeV}, 2.36\text{ TeV}, 7\text{ TeV}, \dots$
nucleus-nucleus $\sqrt{s_{NN}} \approx 5.5\text{ TeV}$

ILC (??, 20??)

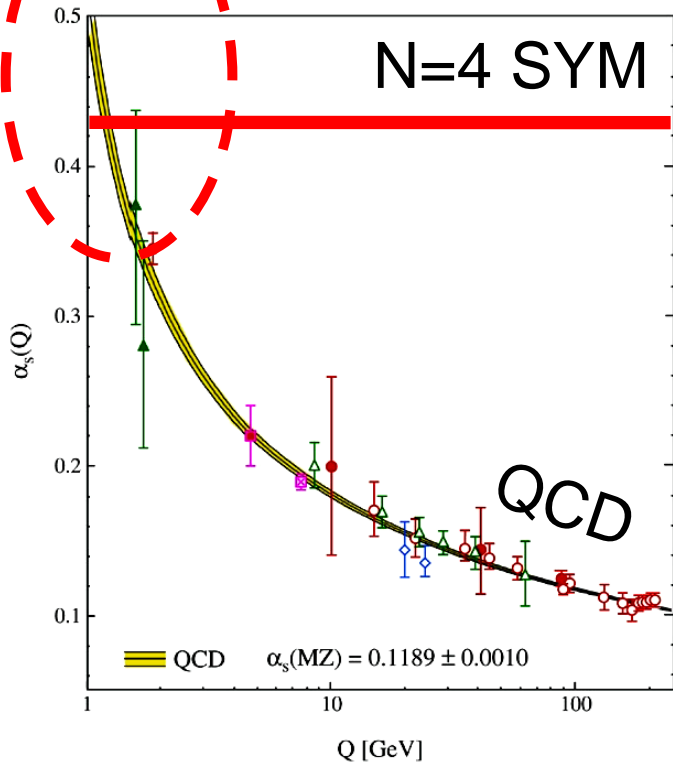
electron—positron annihilation $\sqrt{s} > 500\text{ GeV}$

Processes studied so far

- Hard scattering Polchinski-Strassler
- DIS and Pomeron
Janik-Peschanski; Polchinski-Strassler; Brower-Polchinski-Strassler-Tan
Cornalba-Costa-Penedones; BallonBayona-BoschiFilho-Braga;
AlvarezGaume-Gomez-VasquezMozo; YH-Iancu-Mueller...
- $e+e-$ annihilation
Hofman-Maldacena; YH-Iancu-Mueller; Evans-Tedder
Chesler-Jensen-Karch; YH-Matsuo; Csaki-Reece-Terning...
- Nucleus-nucleus collision
Shuryak-Lin, Nastase, Albacete-Kovchegov-Talotis; Gubser-Pufu-Yarom...
- Polarized DIS Gao-Xiao, YH-Ueda-Xiao...
- Odderon Brower-Djuric-Tan, Avsar-YH-Matsuo
- Drell-Yan BallonBayona-BoschiFilho-Braga

N=4 supersymmetric Yang-Mills

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} + \sum_{i=1}^4 \bar{\psi}_\alpha^i (\bar{\sigma} \cdot D)_{\alpha\beta} \psi_\beta^i + 2\sqrt{2} g f^{abc} \sum_{1 \leq i < j \leq 4} \text{Re} (\phi_a^{ij} \psi_b^{i\alpha} \psi_{c\alpha}^j) \\ + \frac{1}{2} \sum_{1 \leq i < j \leq 4} (D_\mu \phi^{ij})^\dagger D^\mu \phi^{ij} - \frac{g^2}{4} \sum_{\substack{1 \leq i < j \leq 4 \\ 1 \leq k < l \leq 4}} |f_{abc} \phi_b^{ij} \phi_c^{kl}|^2$$



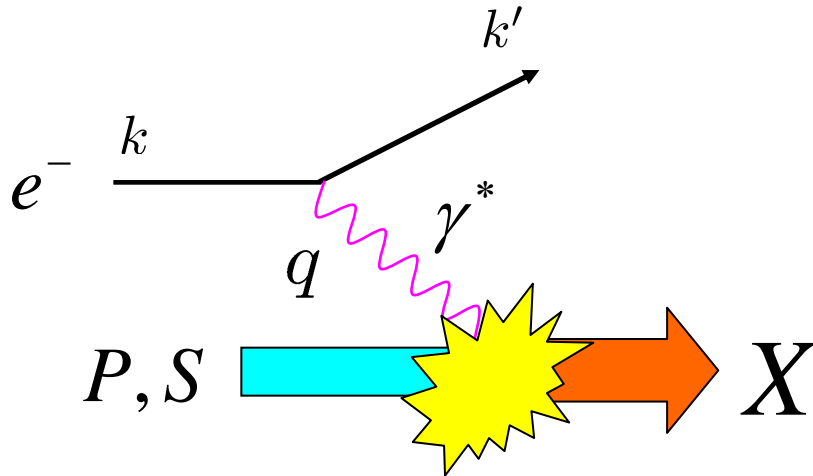
Gauge boson (“gluon”),
4 Weyl fermions (“quarks”),
6 scalars, all in the adjoint rep.

Global SU(4) R-symmetry

Dual to type-IIB superstring on

$$AdS_5 \times S^5$$

Deep inelastic scattering



Photon virtuality

$$q^2 = -Q^2 < 0 \quad (\text{spacelike})$$

Bjorken variable

$$x = \frac{Q^2}{2P \cdot q}$$

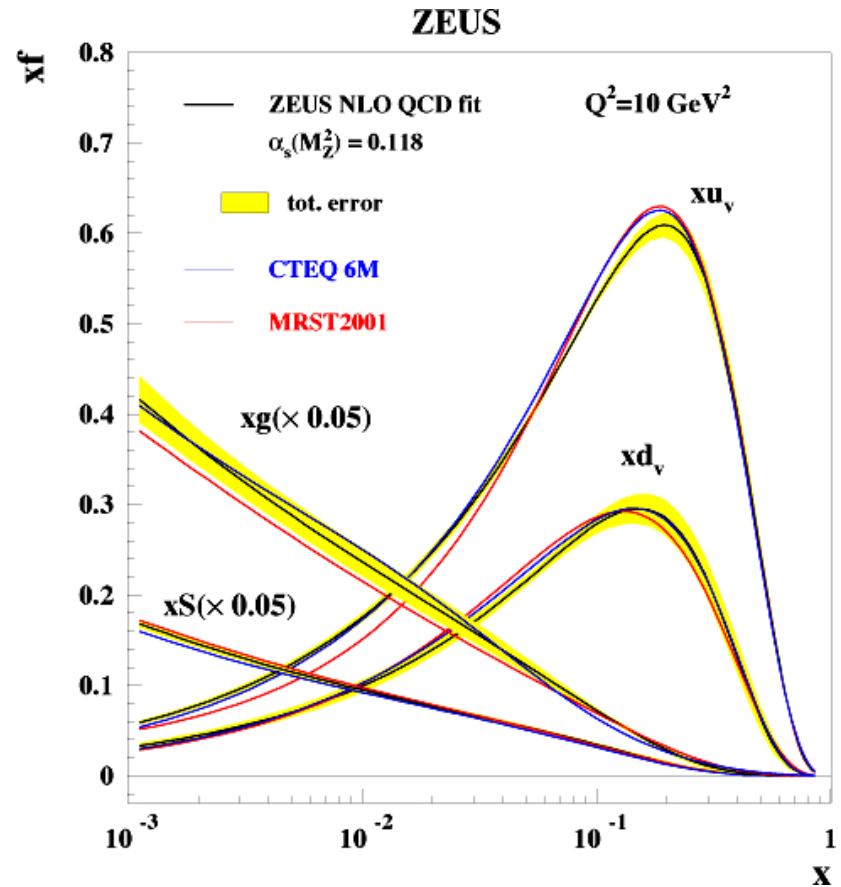
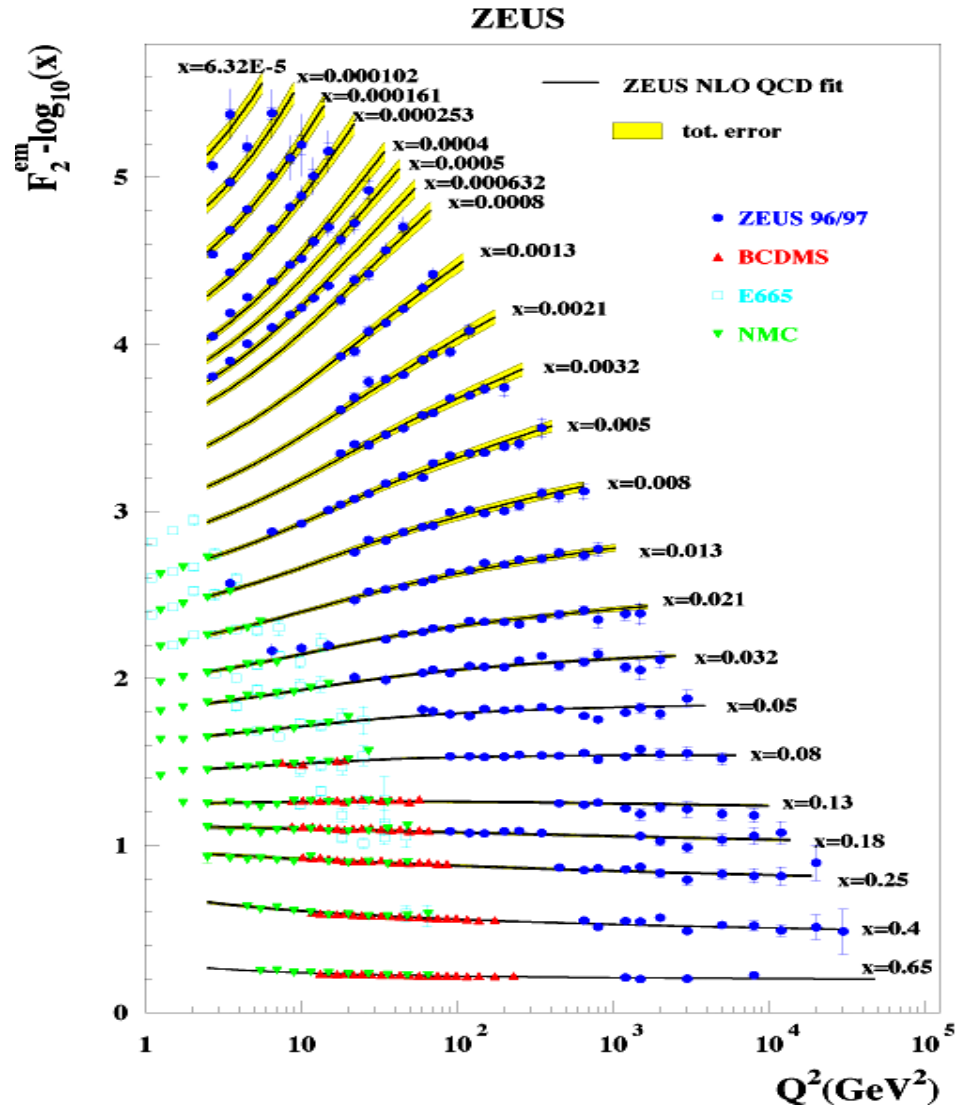
$$\approx \frac{Q^2}{s} \quad (x \ll 1)$$

$$\frac{1}{4\pi} \int d^4y e^{iqy} \langle P, S | [J^\mu(y), J^\nu(0)] | P, S \rangle$$

$$= \left(-\eta^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) F_1(x, Q^2) + \left(P^\mu - \frac{P \cdot q}{q^2} q^\mu \right) \left(P^\nu - \frac{P \cdot q}{q^2} q^\nu \right) \frac{F_2(x, Q^2)}{P \cdot q}$$

$$+ i\epsilon^{\mu\nu\alpha\beta} q_\alpha \left(\frac{S_\beta}{P \cdot q} (g_1(x, Q^2) + g_2(x, Q^2)) - \frac{q \cdot S P_\beta}{(P \cdot q)^2} g_2(x, Q^2) \right)$$

Parton evolution



Hard Pomeron

$$\int_0^1 dx x^{j-2} F_2(x, Q^2) \sim \left(\frac{Q^2}{\mu^2} \right)^{\gamma(j)}$$

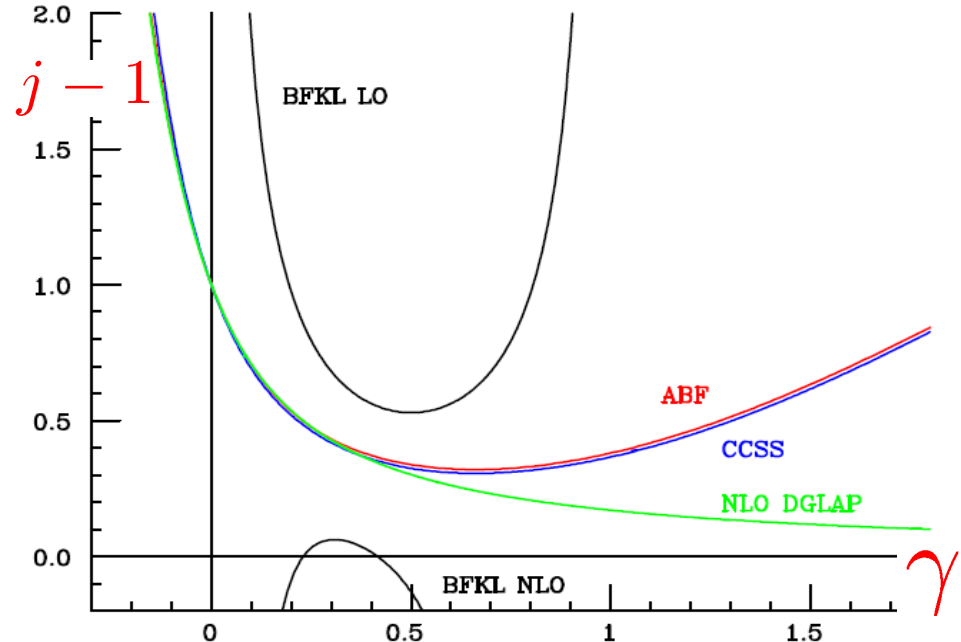
← Anomalous dim. of the twist-two operators

$$\bar{q}_f(0) \gamma^+ (iD^+)^{j-1} q_f(0), \dots$$

$$F_2(x, Q^2)$$

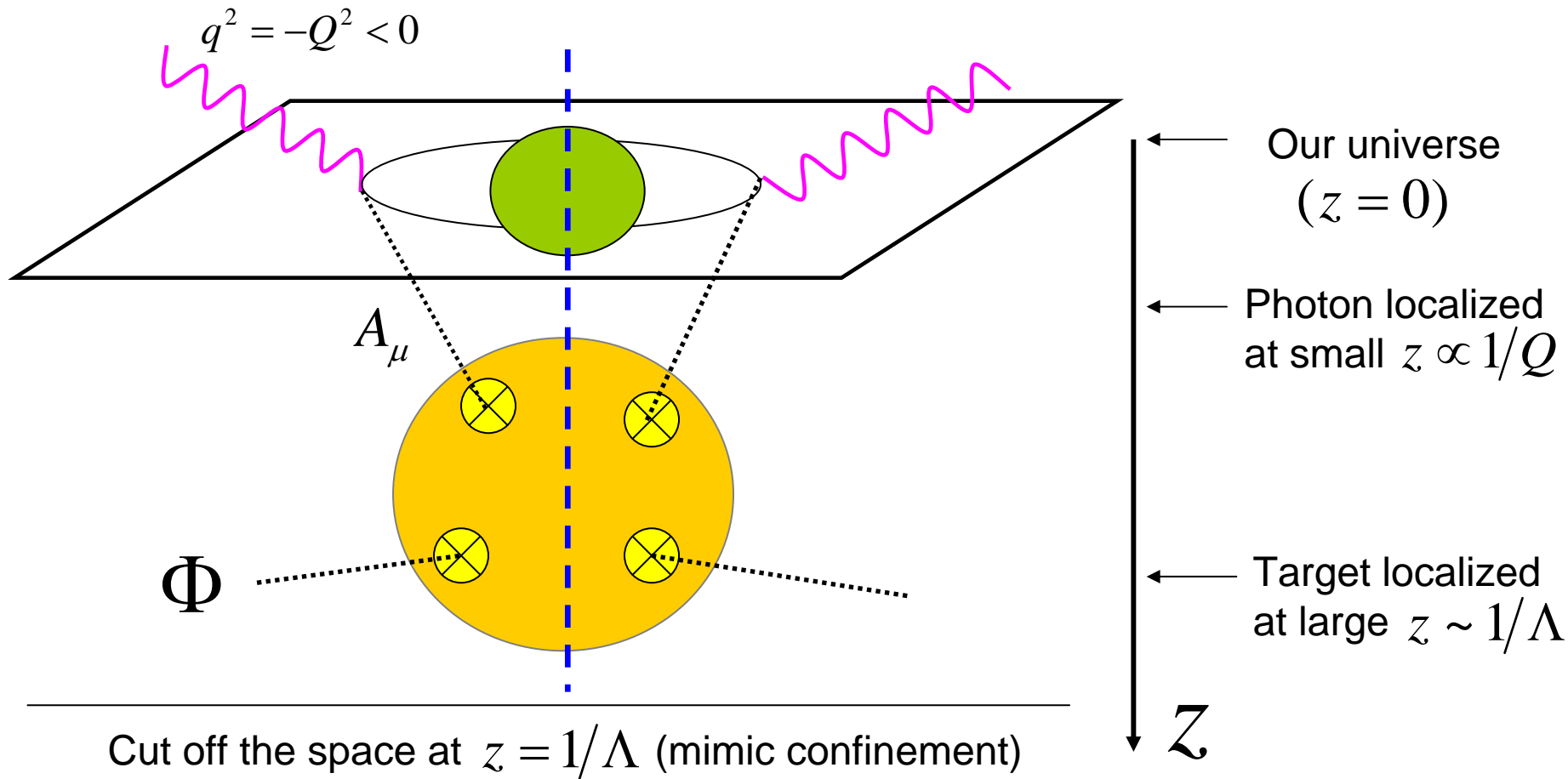
$$\sim \int \frac{dj}{2\pi i} \left(\frac{Q^2}{\mu^2} \right)^{\gamma(j)} \left(\frac{1}{x} \right)^{j-1}$$

Analytically continued to non-integer j



DIS at strong coupling

Polchinski-Strassler (2002)
YH-Iancu-Mueller (2007)



Large-x: No partons !

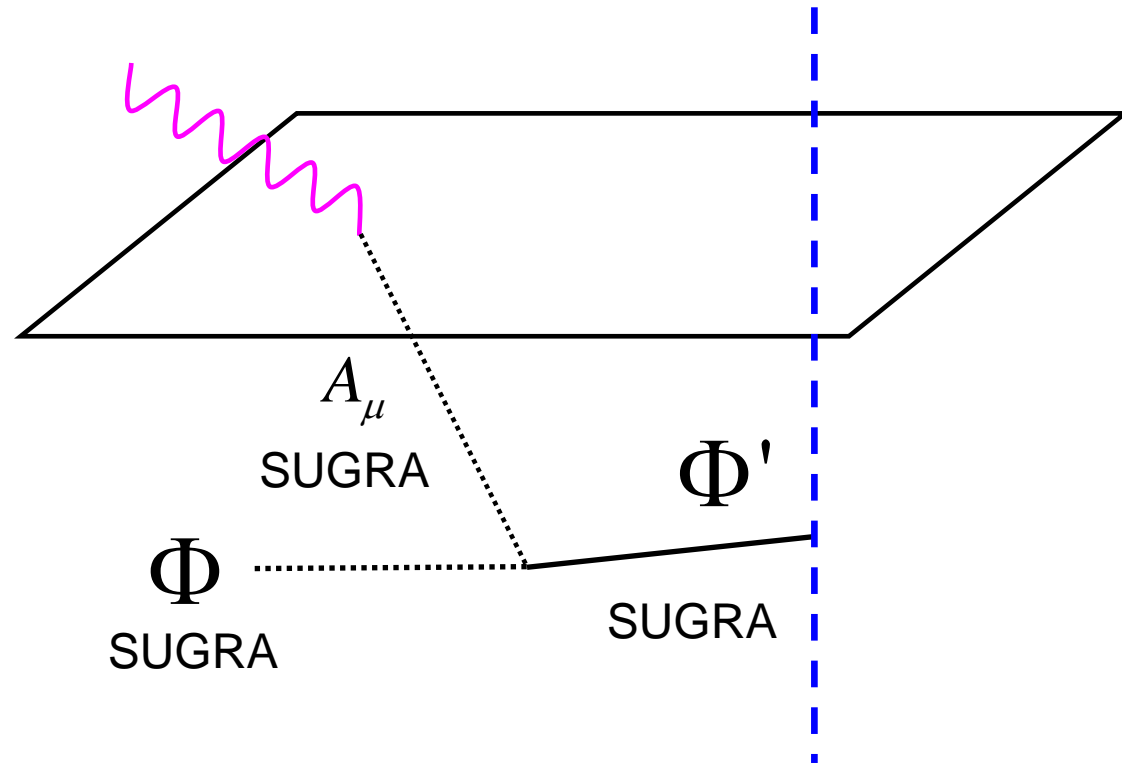
Polchinski, Strassler (2002)

At large-x, a hadron scatters as a whole.

$$s_{4D} = \frac{Q^2}{x}$$

$$s_{5D} = \frac{R^2}{r_{int}^2} s_{4D} < \frac{1}{\alpha'}$$

→ $\frac{1}{\sqrt{\lambda}} < x < 1$



So different from QCD.
Phenomenology? No way !

Small-x: Regge regime

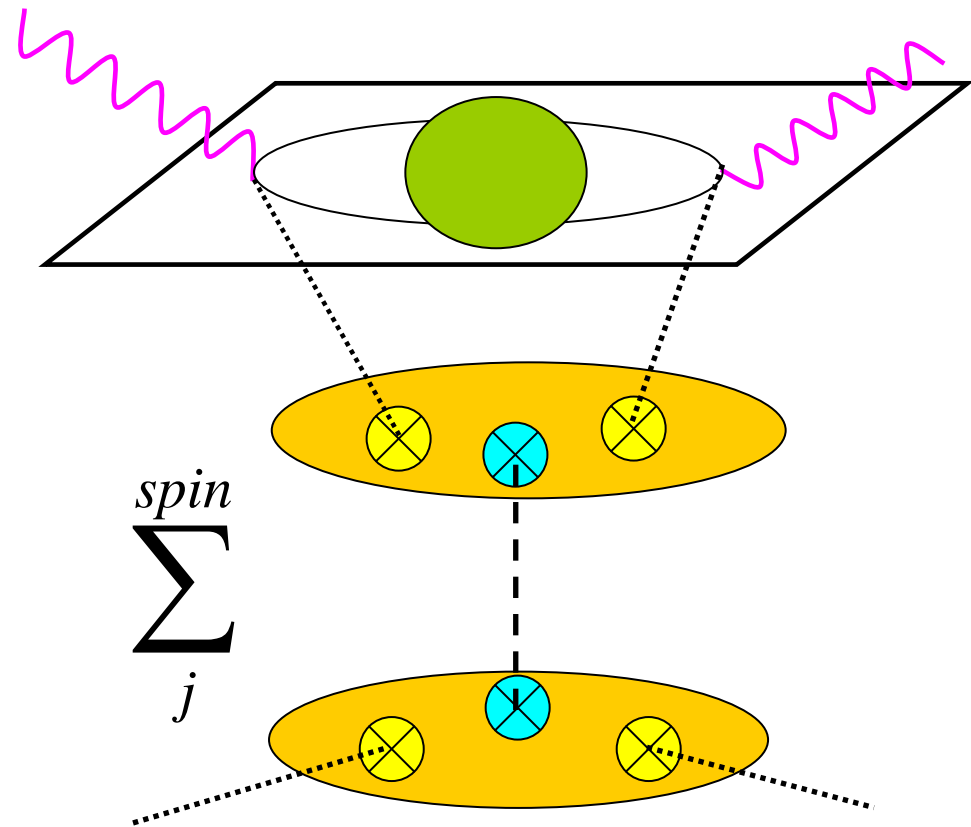
Brower, Polchinski, Strassler, Tan (2006)

$$A \sim s^{2 + \frac{\alpha' t}{2}}$$

$$= \int \frac{dj}{2\pi i} \frac{s^j}{j - 2 - t/2\sqrt{\lambda}}$$

$$t_{5D} = t_{\perp} + t_z$$

Laplacian operator acting on the string exchanged in the t-channel

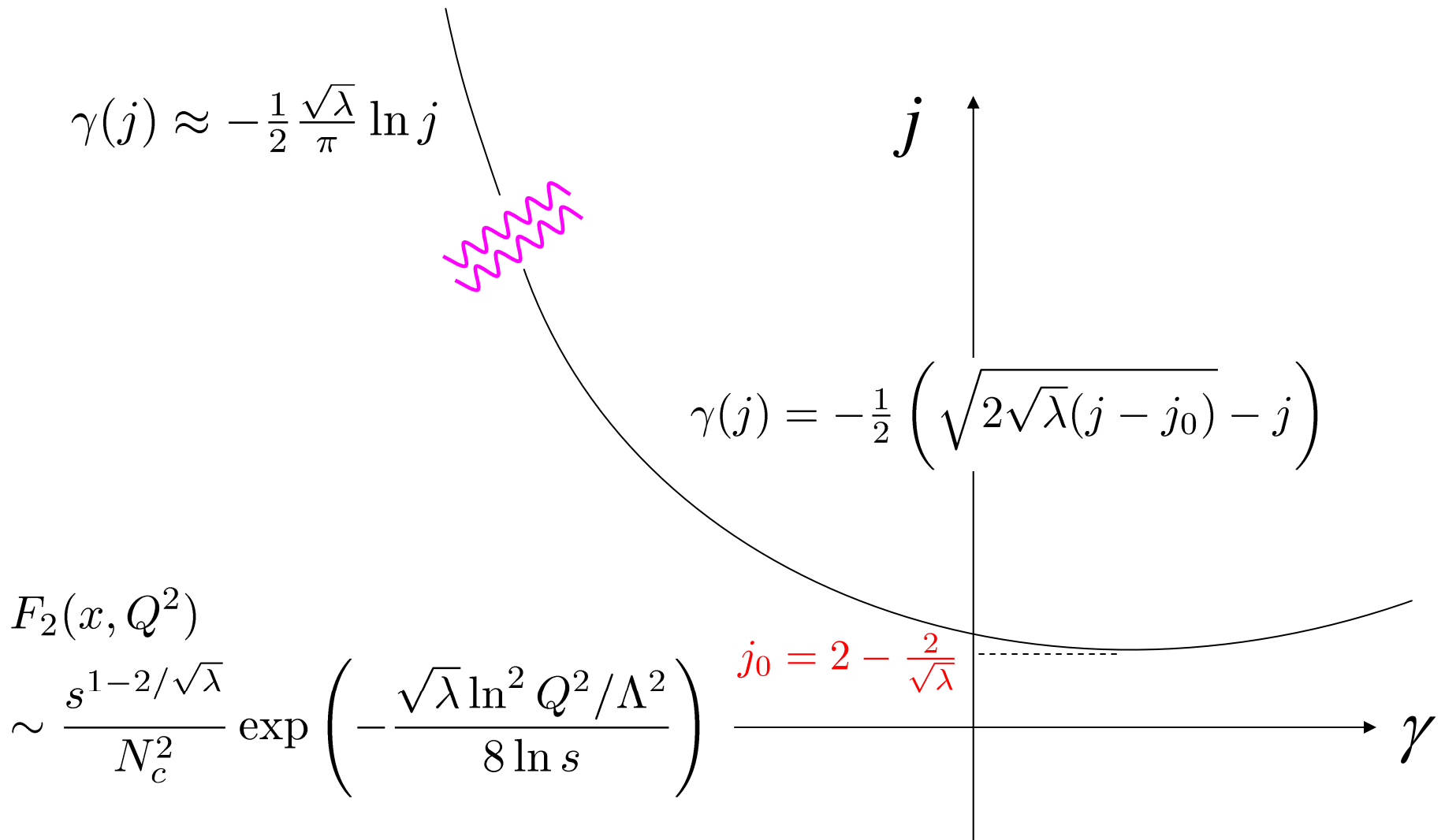


Eigenvalue of t_z shifts the intercept

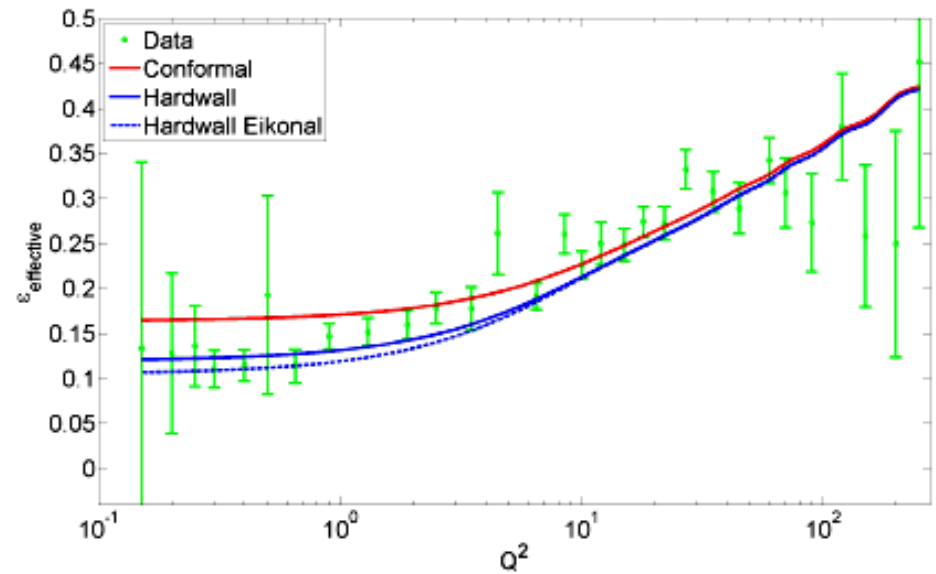
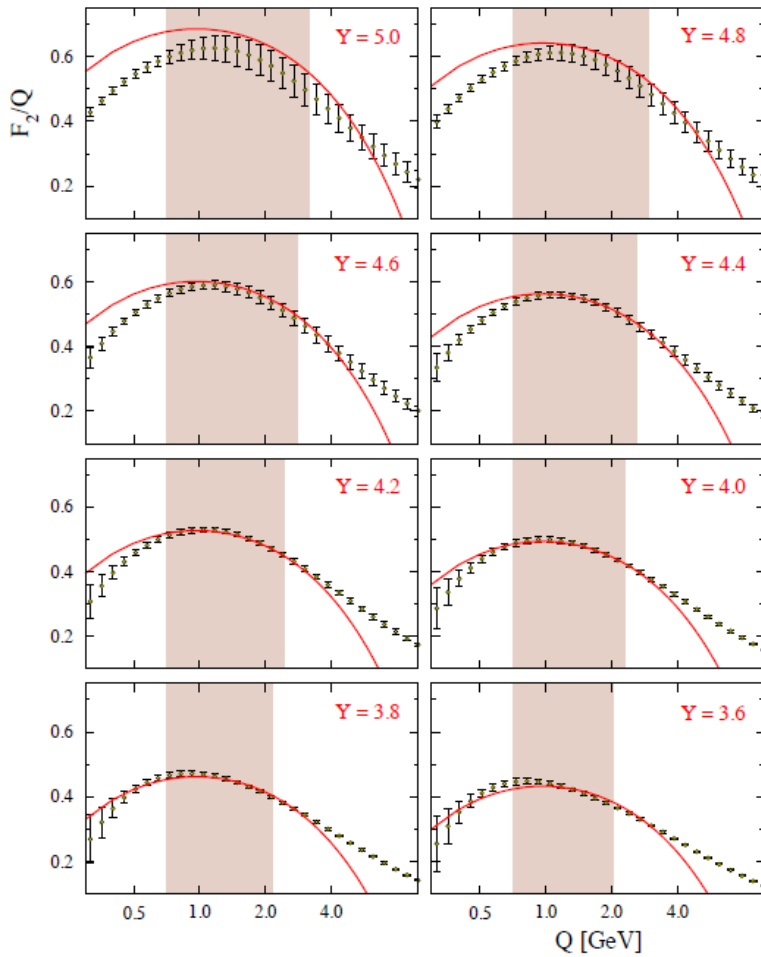
$$j_0 = 2 + \mathcal{O}(1/\sqrt{\lambda})$$

Pomeron= Reggeized graviton in AdS

The anomalous dimension at strong coupling



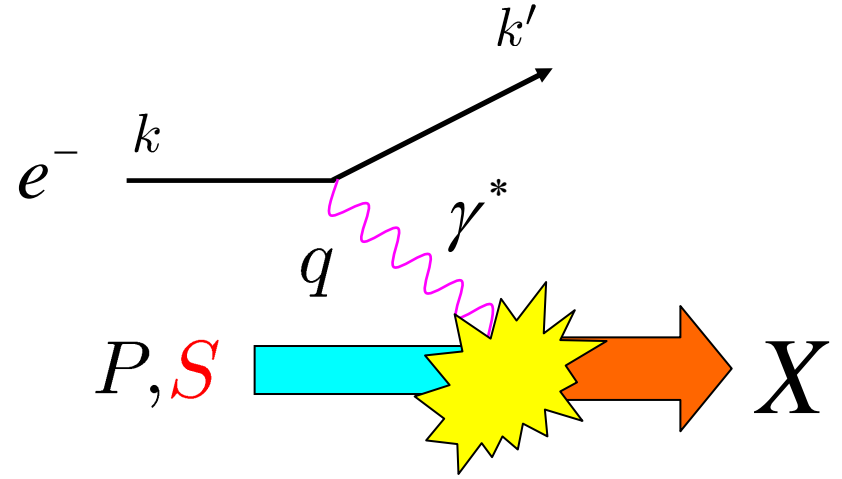
Fits to the HERA data



$$F_2(x, Q^2) \sim \left(\frac{1}{x}\right)^{\epsilon_{\text{eff}}(Q^2)}$$

Polarized DIS

DIS on a **longitudinally**
polarized proton



$$\frac{1}{4\pi} \int d^4y e^{iqy} \langle P, S | [J^\mu(y), J^\nu(0)] | P, S \rangle$$

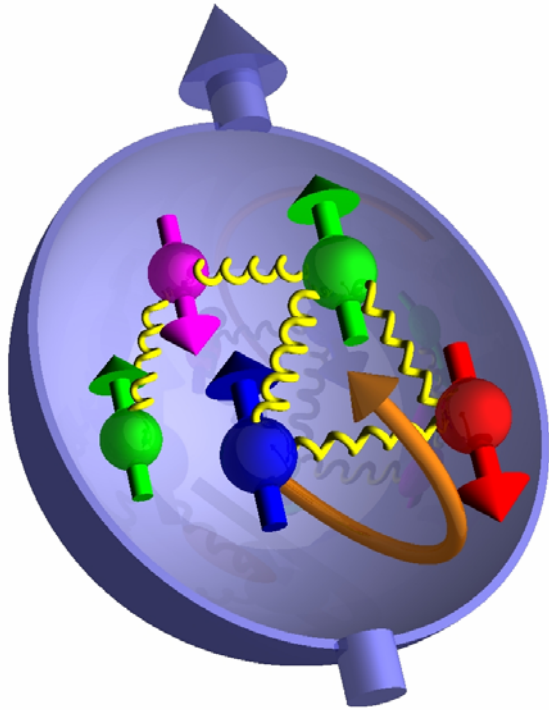
$$\sim i\epsilon^{\mu\nu\alpha\beta} q_\alpha \left(\frac{S_\beta}{P \cdot q} (g_1(x, Q^2) + g_2(x, Q^2)) - \frac{q \cdot S P_\beta}{(P \cdot q)^2} g_2(x, Q^2) \right)$$

$$\int_0^1 dx g_1(x, Q^2) = \frac{1}{4S^+} \sum_f e_f^2 \langle PS | \bar{q}_f \gamma_5 \gamma^+ q_f | PS \rangle = \frac{1}{9} \Delta\Sigma + \frac{1}{12} a^{(3)} + \frac{1}{36} a^{(8)}$$

Quarks' helicity contribution to the proton spin

$$\Delta\Sigma = 1 \quad \text{in the naïve quark model}$$

Spin crisis



Angular momentum decomposition in QCD

$$\frac{1}{2} = \frac{1}{2} \Delta\Sigma + \Delta G + L_z$$

Quarks' helicity

Gluons' helicity

Orbital angular momentum

The EMC shock (1987)

$$\Delta\Sigma = 0.12 \pm 0.09 \pm 0.14 \quad !?$$

Polarized DIS from AdS/CFT

YH, Ueda, Xiao (2009)

Unpolarized

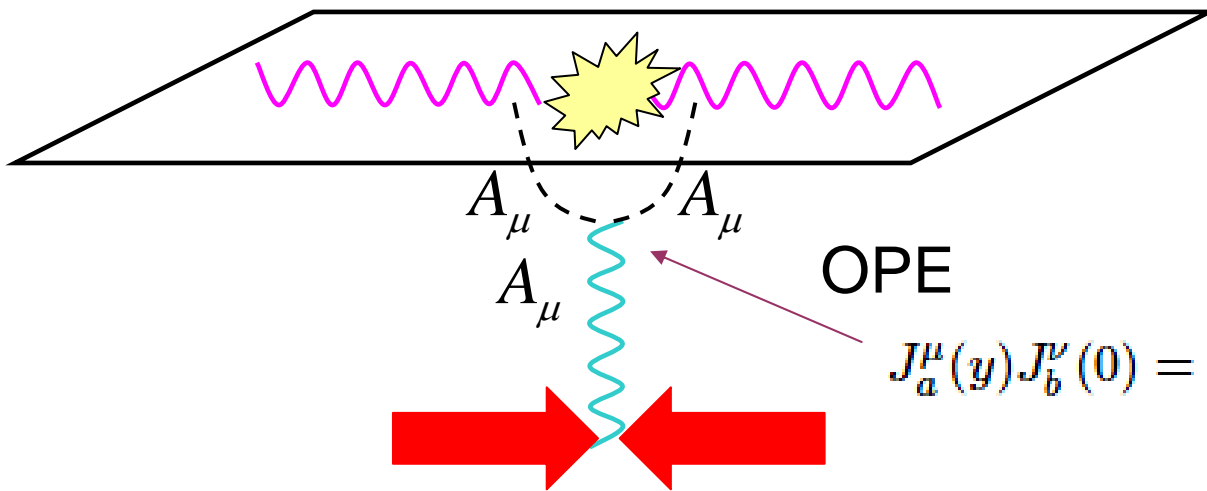
→ graviton exchange $j=2$ $JJ \sim \frac{1}{x^2} T$ $F_1(x, Q^2) \sim \left(\frac{1}{x}\right)^{2-\mathcal{O}(1/\sqrt{\lambda})}$

質問にちゃんと答えられませんでしたでしたが、この仕事では
N=4 SYMのフェルミオンをクォークの代用物として扱っています。

Polarized

→ SO(6) gauge boson exchange $j=1$

$JJ \sim \frac{1}{x} J$ $g_1(x, Q^2), g_2(x, Q^2) \sim \left(\frac{1}{x}\right)^{1-\mathcal{O}(1/\sqrt{\lambda})}$



$$J_a^\mu(y) J_b^\nu(0) = d^{abc} \epsilon^{\mu\nu}{}_{\alpha\beta} \frac{y^\alpha}{3\pi^2 y^4} J_c^\beta(0) + \dots$$

g1 and g2 at strong coupling

$$g_1(x, Q^2)S^+ + \frac{F_3(x, Q^2)}{2}P^+ = \frac{d^{33c} Q_c}{6\pi} \frac{\pi}{4\sqrt{\lambda}} \int d^4y dz \sqrt{G} \int dz' \sqrt{G'} G'^{+-} z z'$$

$$\times \left(\frac{1}{x} \right)^{1 - \frac{1}{2\sqrt{\lambda}}} \frac{e^{-(\rho - \rho')^2 / 4D\tau}}{\sqrt{\pi D\tau}} J_+^{\text{bulk}}(z', y) \bar{\psi} \gamma^+ \psi(z)$$

($\tau = \ln 1/x$)

Sum rule OK.

The g2 structure function is much smaller than g1.

$$g_1(x, Q^2) \sim \frac{c}{x^{1-\epsilon}} \quad \epsilon \sim \mathcal{O}(1/\sqrt{\lambda})$$

→

Wandzura-Wilczek relation

$$g_2(x) \sim -\frac{c}{1-\epsilon} + \frac{\epsilon c}{(1-\epsilon)x^{1-\epsilon}}$$

Comparison with QCD

QCD

N=4 SYM at strong coupling

Experiment

$$\Delta\Sigma \sim 0.3$$

$$\Delta G \approx 0.0 \pm 0.5$$

$$\Delta\Sigma \approx 0$$

$$\Delta G \approx 0 \text{ ??}$$

$$g_1^{singlet}(x) \approx 0$$

Similar

$$g_1^{singlet}(x) \approx 0$$

$$g_1^{non-singlet}(x) \sim \left(\frac{1}{x}\right)^{0.5} ??$$

$$g_1^{non-singlet}(x) \sim \left(\frac{1}{x}\right)^{1-1/2\sqrt{\lambda}}$$

Opposite

pQCD

$$g_1^{singlet}(x) \sim \left(\frac{1}{x}\right)^{2.5 \frac{\sqrt{\lambda}}{2\pi}}$$

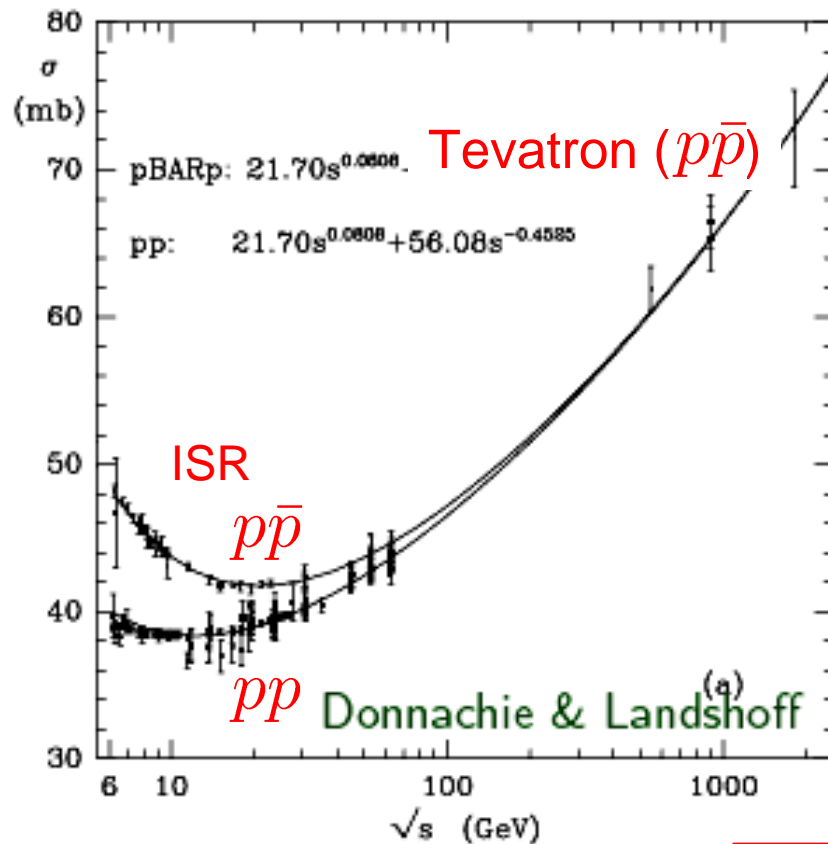
$$g_1^{non-singlet}(x) \sim \left(\frac{1}{x}\right)^{\frac{\sqrt{\lambda}}{2\pi}}$$

Helicity contribution suppressed due to the large anomalous dimension.

Total cross section difference

Consider the difference

$$\Delta\sigma \equiv \sigma_{tot}^{p\bar{p}} - \sigma_{tot}^{pp}$$



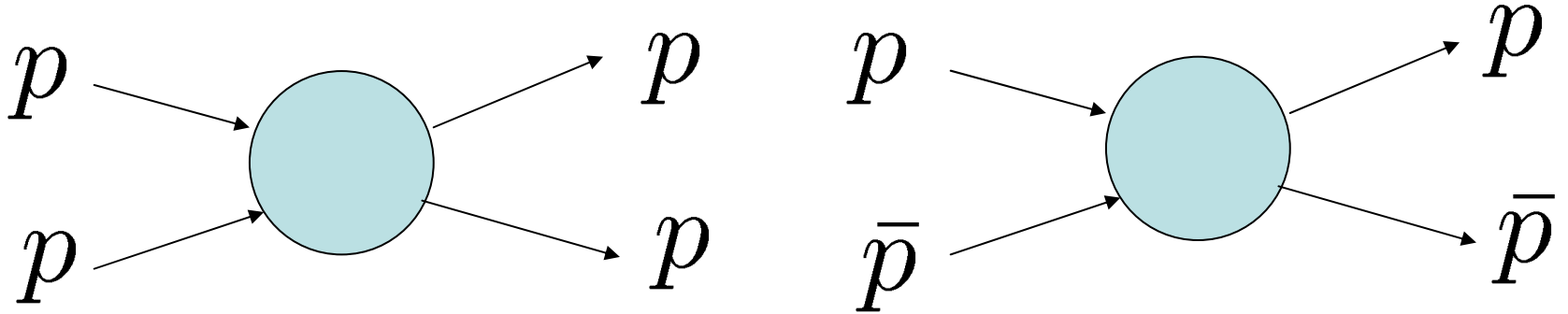
Up to the ISR energy, ($\sqrt{s} = 53\text{GeV}$)

$$\Delta\sigma \sim s^{-0.5} > 0$$

What's the fate of $\Delta\sigma$ as $s \rightarrow \infty$??

→
LHC(pp), cosmic rays

Odderon and Reggeon



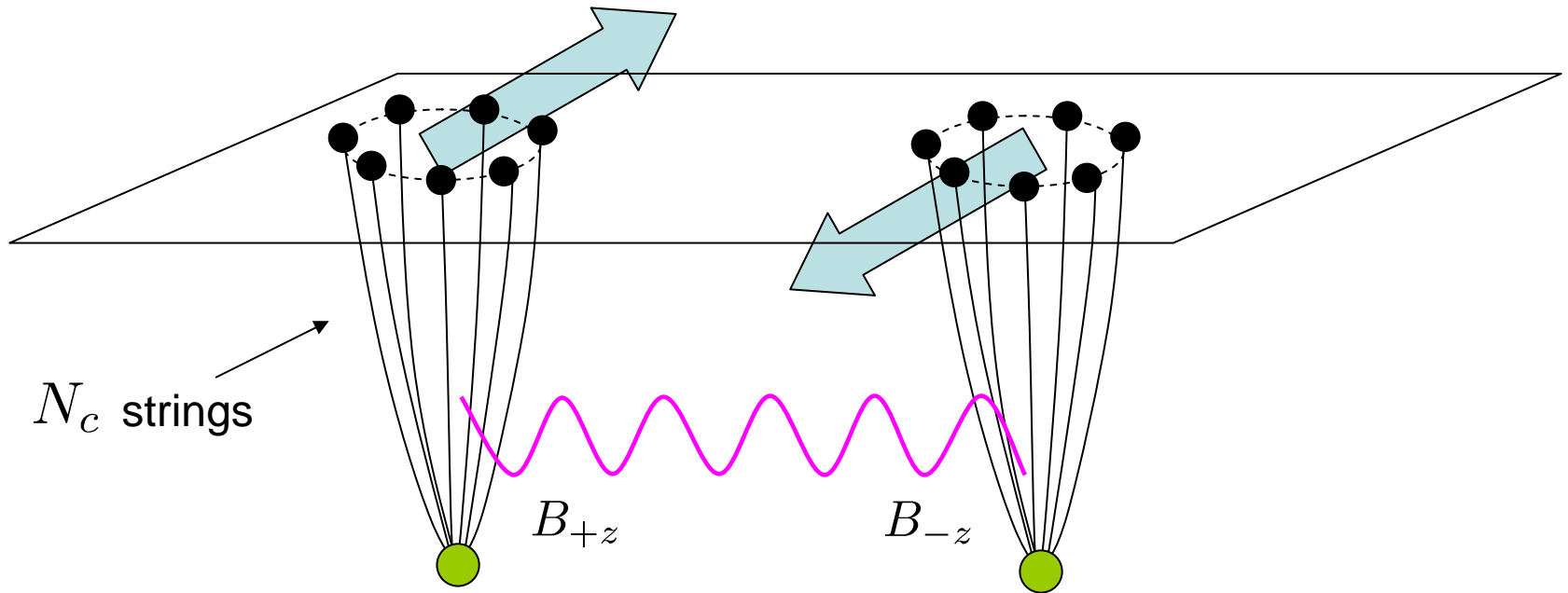
The difference $\Delta\mathcal{A} \equiv \mathcal{A}_{pp \rightarrow pp}(s, t) - \mathcal{A}_{p\bar{p} \rightarrow p\bar{p}}(s, t)$

is odd under crossing, generated by the exchange of **C-odd** objects in QCD

\longrightarrow
 $\left\{ \begin{array}{l} \text{Reggeon (vector mesons)} \\ \text{Odderon (C-odd glueballs)} \end{array} \right.$
Lukaszuk, Nicolescu (1973)

$$\Delta\sigma(s) = \frac{1}{s} \text{Im}\Delta\mathcal{A}(s, t=0) \sim s^{-0.5}$$

Odderon = antisymmetric B-field



N_c strings

B_{+z}

B_{-z}

Baryons as wrapped D-branes

Witten (1998)

Imamura (1999)

B-field couples to the electric field in the D-brane worldvolume

Opposite sign in pp and pbar scattering.

Total cross section difference

Avsar, YH, Matsuo (2009)

$$\begin{aligned}\Delta\sigma &= \sigma^{BB} - \sigma^{\bar{B}\bar{B}} = 2 \int d^2b \operatorname{Im} \mathcal{A}^-(s, b) - 2 \int d^2b \operatorname{Im} \mathcal{A}^+(s, b) \\ &= -\frac{\pi\sqrt{\lambda}}{4(\operatorname{Vol}_{S^4})^2} \sum_{I,k} \frac{M_I + \frac{1}{M_I}}{k+2} \int dz d\Omega_4 Y^{(k)}(\Omega_5) \int dz' d\Omega'_4 Y^{(k)}(\Omega'_5) \left(\frac{zz's}{4\sqrt{\lambda}}\right)^{\alpha_O(0)-1}.\end{aligned}$$

$$\alpha_O(0) = 1 - \frac{M^2 - 1}{2\sqrt{\lambda}} \quad M = 1, 2, \dots$$

$\Delta\sigma$ is **negative** ! ...in conflict with the ISR data

sign of $\Delta\sigma \rightarrow$ sign of $\operatorname{Im}\mathcal{A} \rightarrow$ sign of the interaction
exchange of the B-field \rightarrow **repulsion** $\rightarrow \Delta\sigma > 0$??

However, this may not be true in a **curved** space !!

A prediction

Reggeon exchange gives a **positive** contribution

$$\Delta\sigma \sim s^{\alpha_R - 1} > 0 \quad \alpha_R = 1 - \frac{9}{2\sqrt{\lambda}}, \quad 1 - \frac{16}{2\sqrt{\lambda}}, \dots$$

Odderon exchange gives a **negative** contribution

$$\Delta\sigma \sim s^{\alpha_O - 1} < 0 \quad \alpha_O = 1, \quad 1 - \frac{3}{2\sqrt{\lambda}}, \quad 1 - \frac{8}{2\sqrt{\lambda}}, \dots$$

At the ISR energies, the Reggeon dominates.

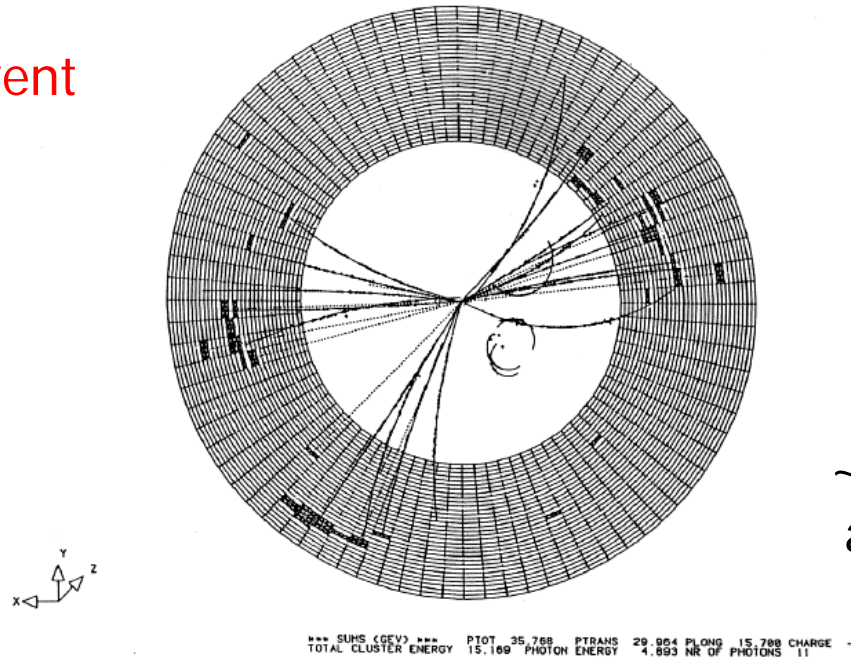
But the Odderon eventually takes over, possibly at the LHC!

オデロンが見えてくるのは $\ln s > \sqrt{\lambda}$ の高いエネルギー領域であり、LHCではこの条件は満たされていると考えられます。

Jets

Observation of jets in 1975 has provided one of the most striking confirmations of QCD

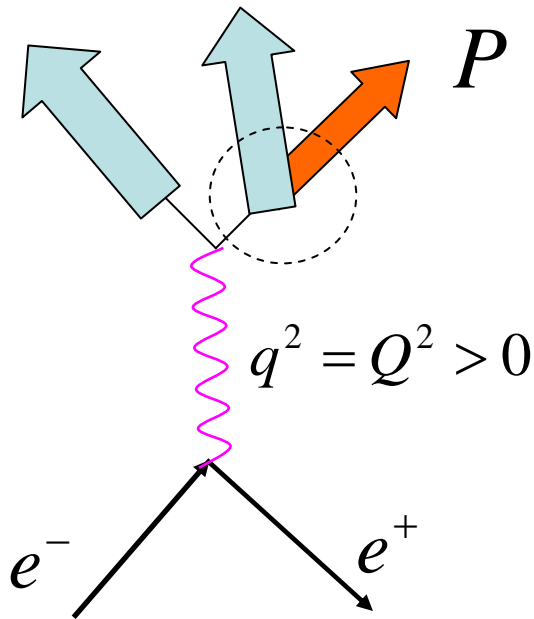
A three-jet event



~90% of the final states
are two-jet events

Average angular distribution $1 + \cos^2 \theta$
reflecting fermionic degrees of freedom

Fragmentation function



Feynman-x

$$x \equiv \frac{2P \cdot q}{Q^2} = \frac{2E}{Q}$$

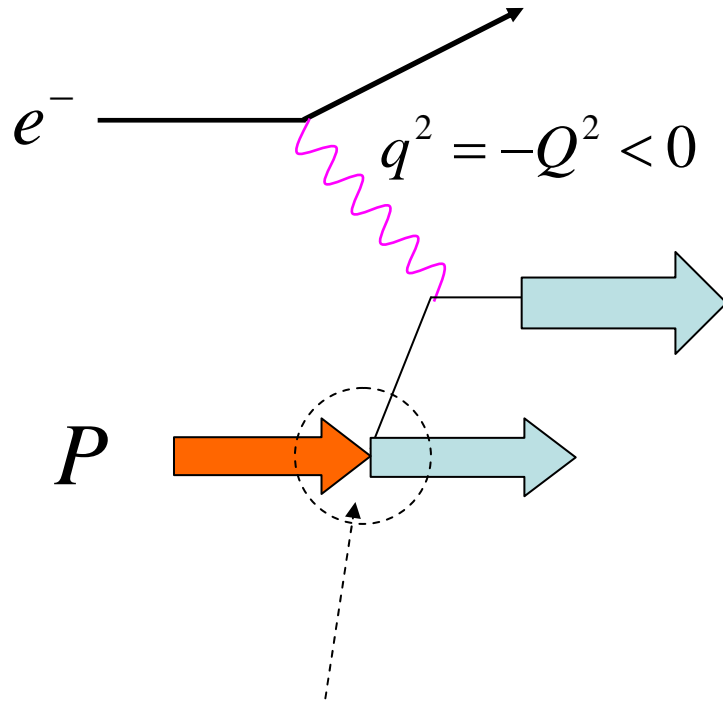
$$\frac{1}{\sigma_{tot}} \frac{d\sigma}{dx} \sim D_T(x, Q^2)$$

Count how many hadrons are there inside a quark.

$$\frac{\partial}{\partial \ln Q^2} D_T(j, Q^2) = \gamma_T(j) D_T(j, Q^2)$$

Timelike anomalous dimension

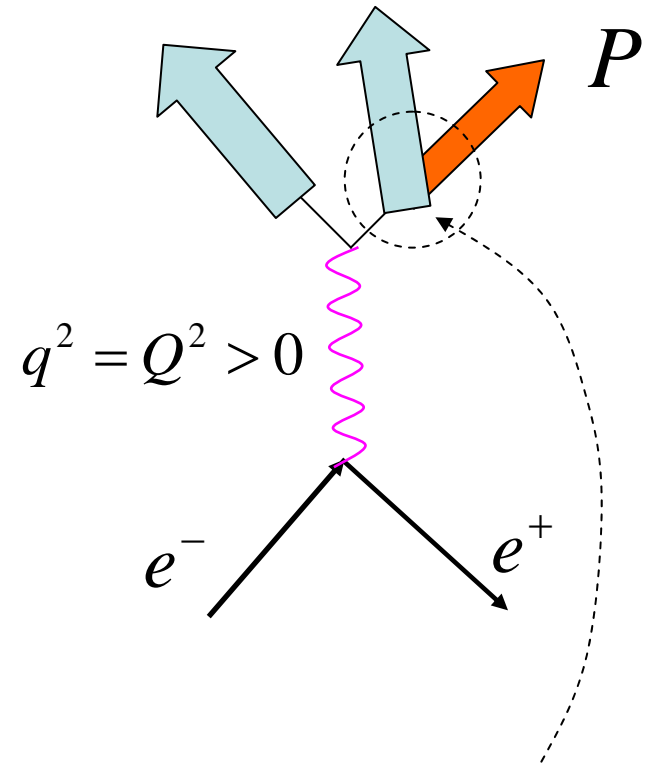
DIS vs. e^+e^- : crossing symmetry



Parton distribution function

$$D_S(x_B, Q^2)$$

Bjorken variable $x_B \equiv \frac{Q^2}{2P \cdot q}$



Fragmentation function

$$D_T(x_F, Q^2)$$

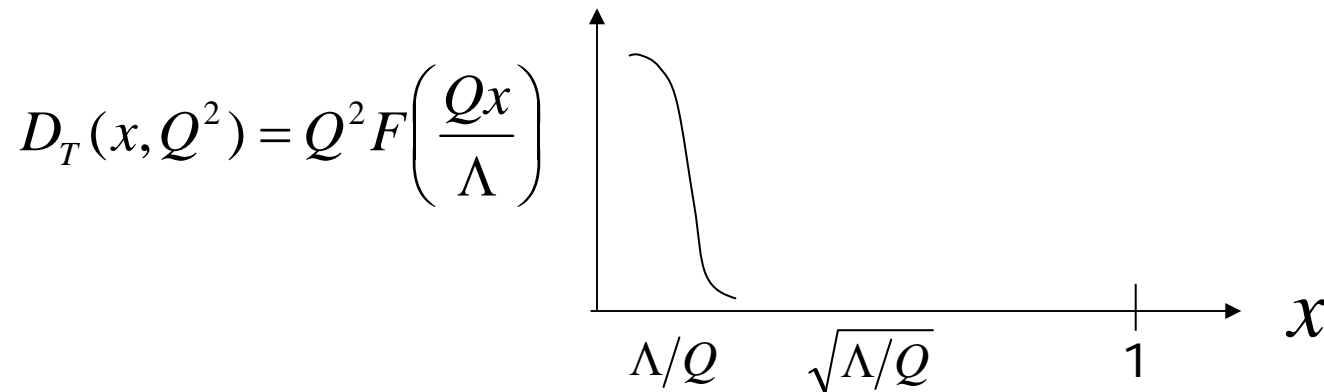
Feynman variable $x_F \equiv \frac{2P \cdot q}{Q^2}$

Fragmentation function at strong coupling

YH, Matsuo (2008)

$$\gamma_S(j) = \frac{j}{2} - \frac{1}{2} \sqrt{2\sqrt{\lambda}(j-j_0)} \quad \xleftrightarrow{\text{crossing}} \quad \gamma_T(j) = -\frac{1}{2} \left(j - j_0 - \frac{j^2}{2\sqrt{\lambda}} \right)$$

$$n(Q) \propto (Q/\Lambda)^{2\gamma_T(1)} = (Q/\Lambda)^{1-3/2\sqrt{\lambda}}$$



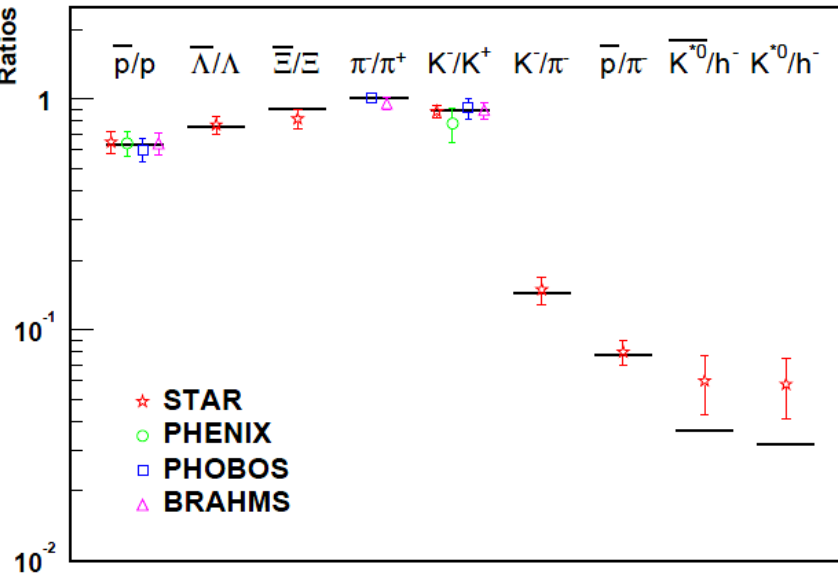
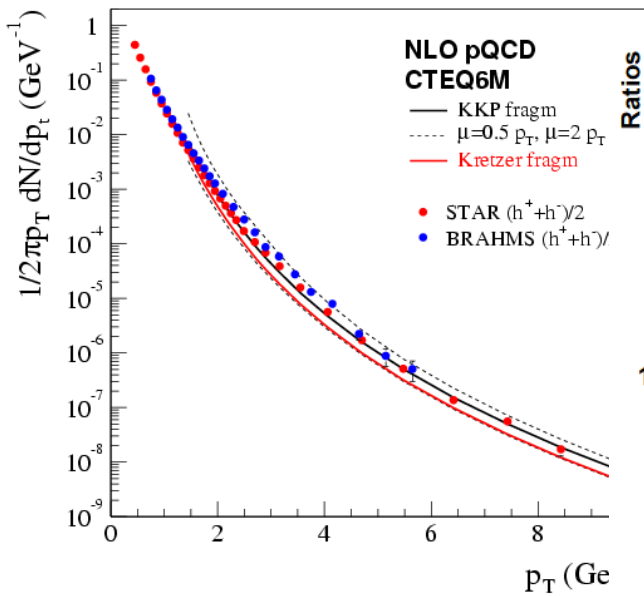
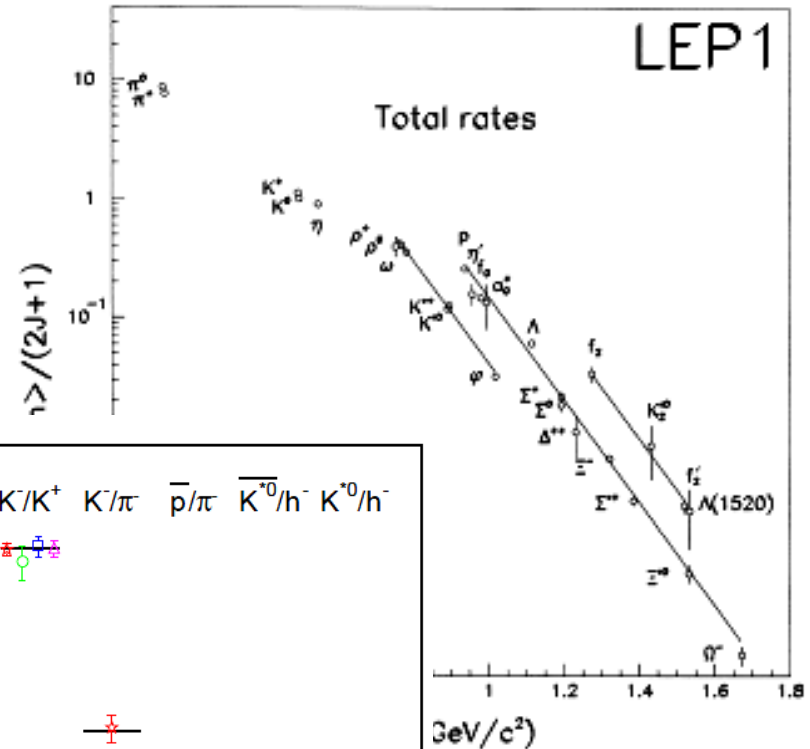
At strong coupling, branching is so fast and complete.
Nothing remains at large- x !

Thermal hadron production

Identified particle yields are well described by a **thermal model**

$$\frac{N^*}{N} \propto \exp\left(-\frac{M^* - M}{T}\right)$$

$$T \sim 170 \text{ MeV}$$



Thermal hadron production from gauge/string duality

$$\begin{aligned}
 & \langle 0 | \epsilon \cdot j(0) | p_1, \dots, p_n \rangle \\
 & \sim \frac{g_c^{n+1}}{\alpha' g_c^2} \int dz d\Omega_5 \sqrt{-GF} (\alpha' \partial^2) (\Phi)^n A_\mu
 \end{aligned}$$

string amplitude
5D hadron w.f.
5D photon

Polchinski, Strassler (2001)

When $n \sim \mathcal{O}(1)$ amplitude dominated by $z_s \sim \frac{1}{p}$

→ Dimensional counting rule by Brodsky, Farrar (1973)

YH, Matsuo (2008)

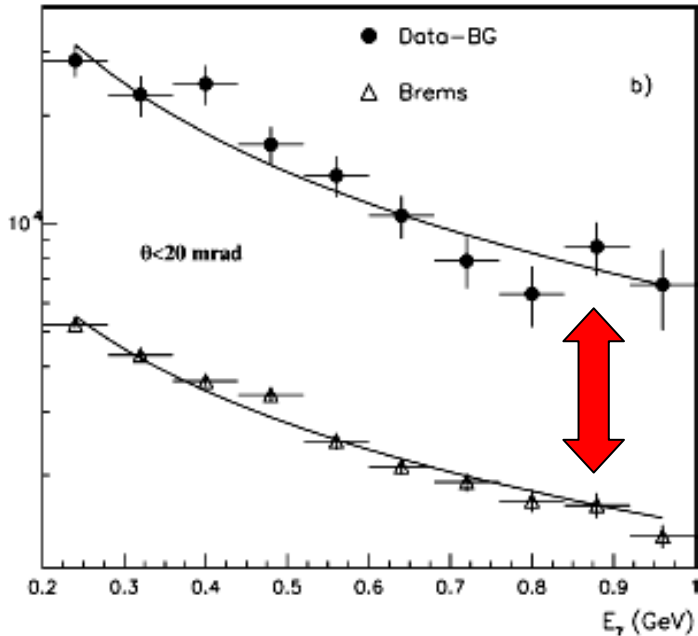
When $n \sim \mathcal{O}(Q)$ and $A_\mu \propto H_1^{(1)}(Qz) \sim e^{iQz}$

Saddle point imaginary $z_s \sim \frac{i}{\Lambda}$ $e^{iQz_s} \sim e^{-Q/\Lambda}$

Thermal !

Soft photon puzzle

YH, Ueda (2010)
T.Ueda (talk yesterday)

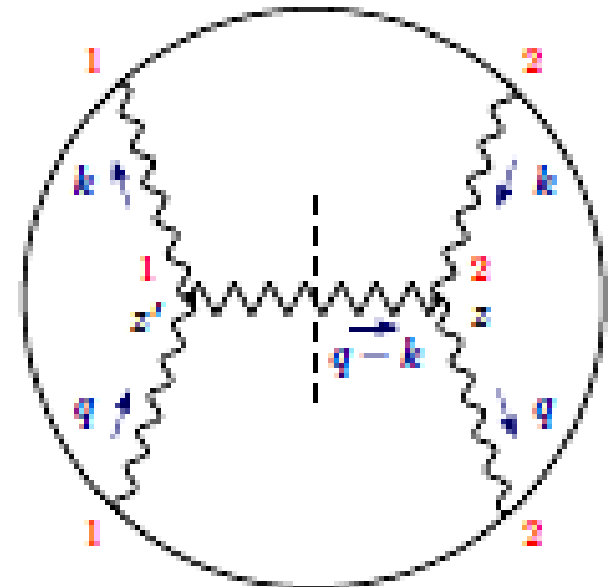


Factor 5 discrepancy between the data and theory (Bremsstrahlung).

Most recent analysis by **DELPHI** (e+e-)

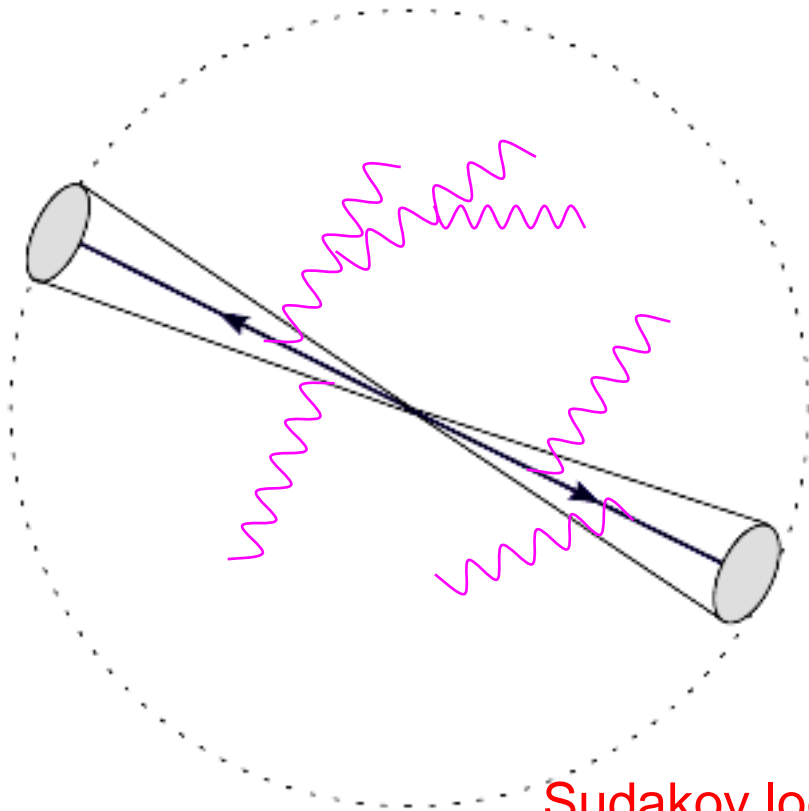
Compute the four-point correlator of the R-current operator from AdS/CFT

$$\frac{dN}{dk} = \frac{\alpha_{em}}{2\pi k} \quad \text{Exact !}$$



Novel source of soft photons

Away-from-jets region



Gluons emitted at large angle,
insensitive to the collinear singularity

Resum only the soft logarithms

$$(\alpha_s \ln 1/x)^n$$

There are **two** types of logarithms.

Sudakov logs. (emission from primary partons)

Kidonakis, Oderda, Sterman (1997)

Non-global logs. (emission from secondary gluons)

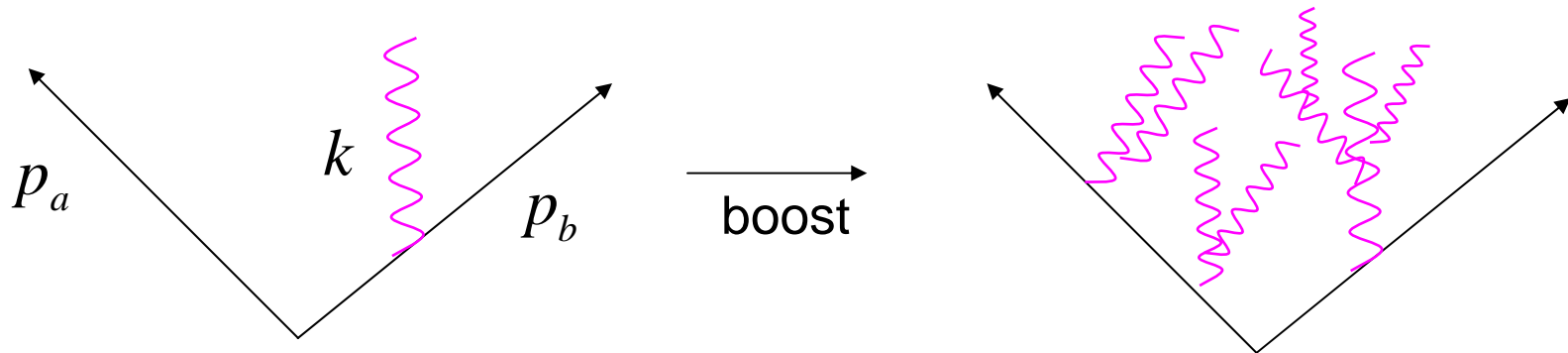
Dasgupta, Salam (2001)

Marchesini-Mueller equation

Marchesini, Mueller (2003)

Differential probability for the soft gluon emission

$$dP = \bar{\alpha}_s \omega d\omega \frac{d\Omega_k}{4\pi} \frac{p_a \cdot p_b}{(p_a \cdot k)(k \cdot p_b)} \approx \bar{\alpha}_s \frac{d\omega}{\omega} \frac{d\Omega_k}{4\pi} \frac{1 - \cos \theta_{ab}}{(1 - \cos \theta_{ak})(1 - \cos \theta_{bk})}$$



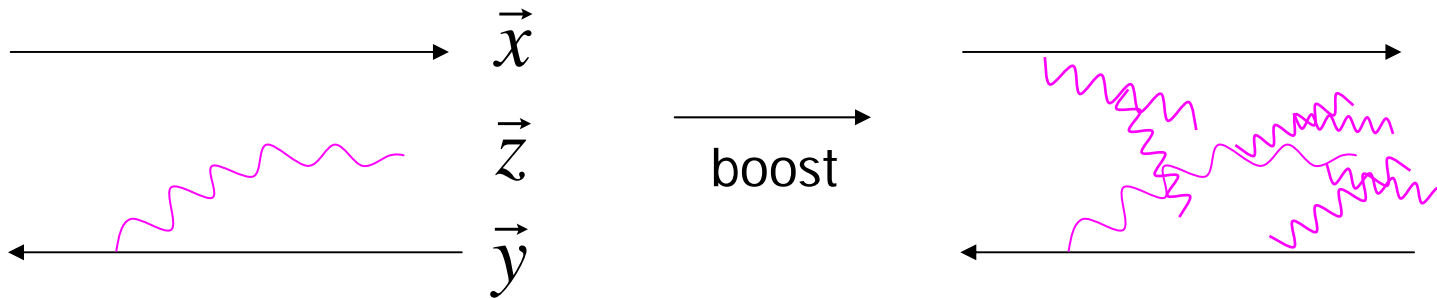
Evolution of the **interjet gluon number**. Non-global logs included.

$$\partial_Y n(\theta_{ab}, \theta_{cd}, Y) = \bar{\alpha}_s \int \frac{d^2\Omega_k}{4\pi} \frac{1 - \cos \theta_{ab}}{(1 - \cos \theta_{ak})(1 - \cos \theta_{bk})} \times (n(\theta_{ak}, \theta_{cd}, Y) + n(\theta_{bk}, \theta_{cd}, Y) - n(\theta_{ab}, \theta_{cd}, Y)). \quad Y = \ln 1/x$$

BFKL equation

Differential probability for the dipole splitting

$$dP = \bar{\alpha}_s \frac{d\omega}{\omega} d^2 \vec{z} \frac{(\vec{x} - \vec{y})^2}{(\vec{x} - \vec{z})^2 (\vec{z} - \vec{y})^2}$$

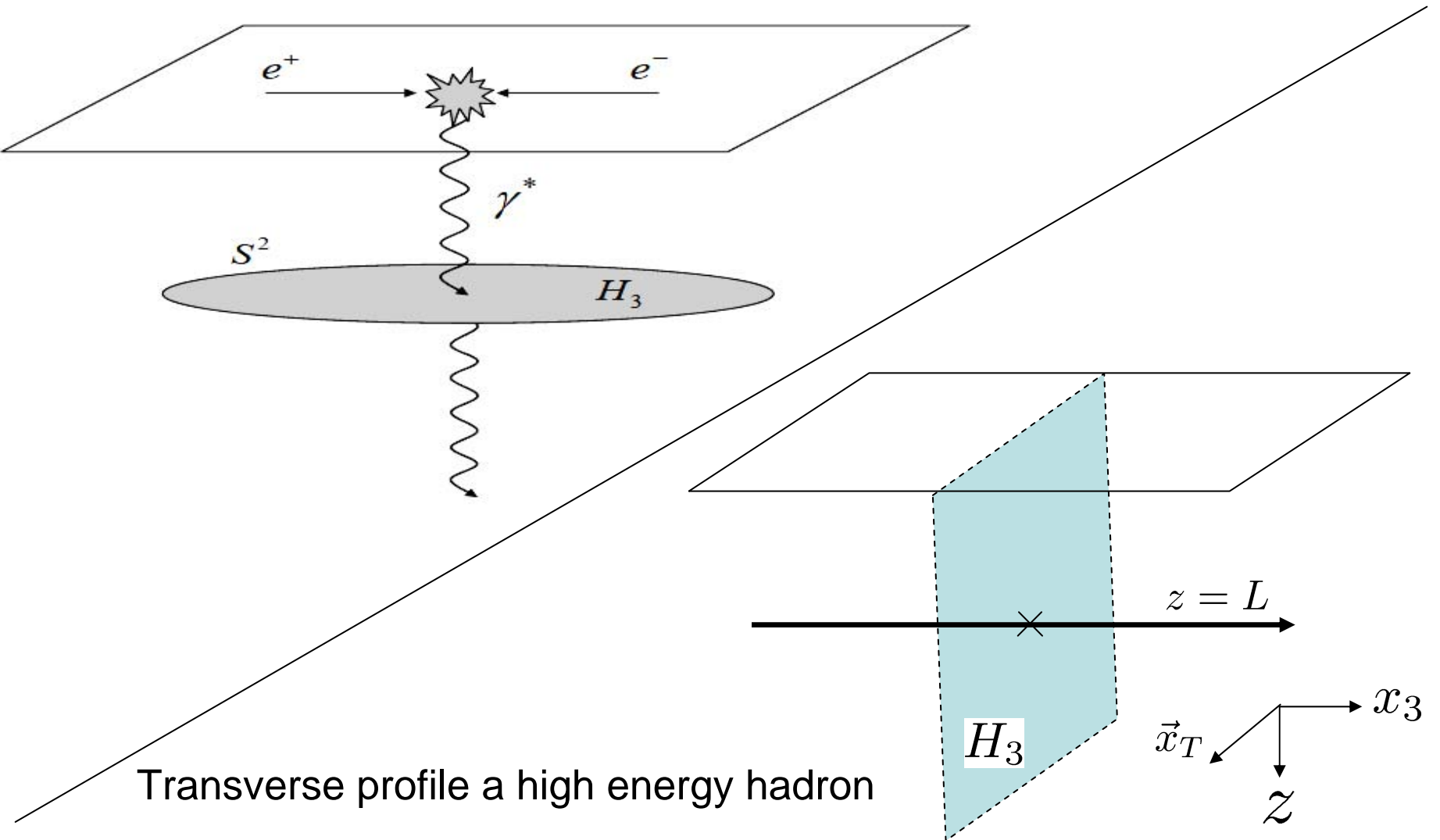


Dipole version of the BFKL equation [Mueller \(1995\)](#)

$$\partial_Y n(x_{ab}, x_{cd}, Y) = \bar{\alpha}_s \int \frac{d^2 \vec{x}_k}{2\pi} \frac{(\vec{x}_{ab})^2}{(\vec{x}_{ak})^2 (\vec{x}_{bk})^2} \times (n(x_{ak}, x_{cd}, Y) + n(x_{bk}, x_{cd}, Y) - n(x_{ab}, x_{cd}, Y))$$

A hint from AdS/CFT

Final state of e^+e^- annihilation



Transverse profile a high energy hadron

The stereographic map

$$\langle \mathcal{E}(\vec{x}_T) \rangle = \frac{2EL^4}{\pi(L^2 + (\vec{x}_T)^2)^3}$$

Gubser, Pufu Yarom (2008)

\vec{x}

$$\langle \mathcal{E}(\Omega) \rangle = \frac{Q}{4\pi}$$

Hofman, Maldacena (2008)

Ω

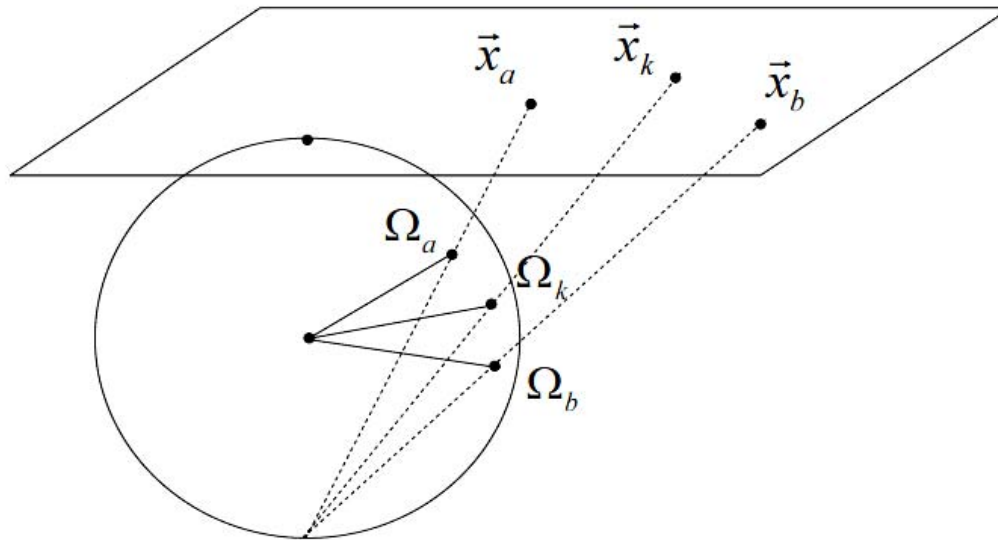
Exact map between the final state in e^+e^- annihilation and the high energy hadronic wavefunction !

YH (2008)

Exact map at weak coupling

The **same** stereographic map transforms BFKL into Marchesini-Mueller

$$\frac{d^2\Omega_k}{4\pi} \frac{1 - \cos\theta_{ab}}{(1 - \cos\theta_{ak})(1 - \cos\theta_{bk})} = \frac{d^2\vec{x}_k}{2\pi} \frac{(\vec{x}_{ab})^2}{(\vec{x}_{ak})^2(\vec{x}_{bk})^2}$$



Make the most of **conformal symmetry $SL(2,C)$** of the BFKL kernel.
Exact solution to the Marchesini—Mueller equation **YH (2008)**
and much more ! **Avsar, YH, Matsuo (2009)**

NLL timelike dipole evolution in N=4 SYM

Avsar, YH, Matsuo (2009)

Apply the stereographic projection
to the result by [Balitsky & Chirilli \(2008\)](#).

$$\partial_Y n_Y(\Omega_{ab}) = \bar{\alpha}_s \left(1 - \bar{\alpha}_s \frac{\pi^2}{12} \right) \int d^2\Omega_c K_{ab}(\Omega_c) [n_Y(\Omega_{ac}) + n_Y(\Omega_{cb}) - n_Y(\Omega_{ab})] \\ + \bar{\alpha}_s^2 \int d^2\Omega_c d^2\Omega_d K'_{ab}(\Omega_c, \Omega_d) n_Y(\Omega_{cd}),$$

$$K'_{ab}(\Omega_c, \Omega_d) = \frac{1}{8\pi^2} \left\{ \frac{(1 - \cos \theta_{ab})}{(1 - \cos \theta_{ac})(1 - \cos \theta_{cd})(1 - \cos \theta_{db})} \right. \\ \times \left[\left(1 + \frac{(1 - \cos \theta_{ab})(1 - \cos \theta_{cd})}{(1 - \cos \theta_{ac})(1 - \cos \theta_{bd}) - (1 - \cos \theta_{ad})(1 - \cos \theta_{bc})} \right) \right. \\ \times \ln \frac{(1 - \cos \theta_{ac})(1 - \cos \theta_{bd})}{(1 - \cos \theta_{ad})(1 - \cos \theta_{bc})} + 2 \ln \frac{(1 - \cos \theta_{ab})(1 - \cos \theta_{cd})}{(1 - \cos \theta_{ad})(1 - \cos \theta_{bc})} \left. \right] \\ \left. + 12\pi^2 \zeta(3) \delta^{(2)}(\Omega_{ac}) \delta^{(2)}(\Omega_{bd}) \right\}.$$

Conclusion

- AdS/CFT offers a new approach to nonperturbative aspects of high energy scattering.
- Some results completely different from QCD, some are useful.
- Feedback to weakly coupled, approximately conformal QCD. Certain features are universal !